Twisted representations of vertex operator algebras and associative algebras

Chongying Dong[1]
Department of Mathematics, University of California, Santa Cruz, CA 95064
Haisheng Li[2]
Department of Mathematical Sciences, Rutgers University, Camden, NJ 08102
Geoffrey Mason[3]
Department of Mathematics, University of California, Santa Cruz, CA 95064

Abstract

Let V be a vertex operator algebra and g an automorphism of order T. We construct a sequence of associative algebras $A_{g,n}(V)$ with $n \in \frac{1}{T}\mathbb{Z}$ nonnegative such that $A_{g,n}(V)$ is a quotient of $A_{g,n+1/T}(V)$ and a pair of functors between the category of $A_{g,n}(V)$-modules which are not $A_{g,n-1/T}(V)$-modules and the category of admissible V-modules. These functors exhibit a bijection between the simple modules in each category. We also show that V is g-rational if and only if all $A_{g,n}(V)$ are finite-dimensional semisimple algebras.

1 Introduction

In this paper we continue our study of twisted representations of vertex operator algebras and lay some further foundations of orbifold conformal field theory.

Given a vertex operator algebra V and an automorphism g of finite order T, we have constructed an associative algebra $A_g(V)$ in [DLM1] such that there is a bijection between simple $A_g(V)$-modules and irreducible admissible g-twisted V-modules, generalizing Zhu's algebra $A(V)$ [Z]. It was proved in [DLM1] that $A_g(V)$ is a finite-dimensional semisimple algebra if V is g-rational. But it is not clear whether the semisimplicity of $A_g(V)$ implies the g-rationality of V. Partially motivated by this in the case $g = 1$ and by the induced module theory for vertex operator algebra we constructed a series of associative algebras $A_n(V)$ for nonnegative integer n in [DLM2] so that $A_{n-1}(V)$ is quotient algebra of $A_n(V)$

[1]Supported by NSF grant DMS-9303374 and a faculty research funds granted by the University of California at Santa Cruz.
[2]Supported by NSF grant DMS-9616630.
[3]Supported by NSF grant DMS-9401272 and a faculty research funds granted by the University of California at Santa Cruz.

induced from the identity map on V and that there is a one to one correspondence between the simple $A_n(V)$-modules which cannot factor through $A_{n-1}(V)$ and irreducible admissible V-modules. Moreover, V is rational if and only if all $A_n(V)$ are finite-dimensional semisimple algebras. In the case $n = 0$ we have Zhu's original algebra $A(V)$.

In this paper we will study the twisted analogues of the algebras $A_n(V)$. In particular, we will construct a series of associative algebras $A_{g,n}(V)$ for nonnegative numbers $n \in \frac{1}{T}\mathbb{Z}$ and show that $A_{g,n-1/T}(V)$ is a natural quotient of $A_{g,n}(V)$. As in the untwisted case, there is a bijection between the simple $A_{g,n}(V)$-modules which cannot factor through $A_{g,n-1/T}(V)$ and the irreducible admissible g-twisted V-modules. In the case $g = 1$ we recover the algebras $A_n(V)$ and the case $n = 0$ amounts to the algebra $A_g(V)$.

Since most results in this paper are similar to those in [DLM2] where $g = 1$, we refer the reader in a lot of places of this paper to [DLM2] for details. We assume that the reader is familiar with the elementary theory of vertex operator algebras as found in [B], [FLM], [FHL] and the definition of twisted modules for vertex operator algebras and related ones as presented in [DLM1].

This paper is organized as follows: In Section 2 we introduce the associative algebras $A_{g,n}(V)$. In Section 3 we construct the functor Ω_n from admissible g-twisted modules to $A_{g,n}(V)$-modules. We show that the each homogeneous subspaces of of the first n pieces is a module for $A_{g,n}(V)$. The Section 4 is the heart of the paper. In this section we construct the functor L_n from $A_{g,n}(V)$-modules to admissible g-twisted V-modules. The main strategy is to prove the associativity for twisted vertex operators.

2 The associative algebra $A_{g,n}(V)$

Let $V = (V, Y, \mathbf{1}, \omega)$ be a vertex operator algebra and g be an automorphism of V of order T. Then V is a direct sum of eigenspaces of T :

$$V = \bigoplus_{r=0}^{T-1} V^r \tag{2.1}$$

where $V^r = \{v \in V | gv = e^{-2\pi ir/T}v\}$.

Fix $n = l + \frac{i}{T} \in \frac{1}{T}\mathbb{Z}$ with l a nonnegative integer and $0 \le i \le T-1$. For $0 \le r \le T-1$ we define $\delta_i(r) = 1$ if $i \ge r$ and $\delta_i(r) = 0$ if $i < r$. We also set $\delta_i(T) = 1$. Let $O_{g,n}(V)$ be the linear span of all $u \circ_{g,n} v$ and $L(-1)u + L(0)u$ where for homogeneous $u \in V^r$ and $v \in V$,

$$u \circ_{g,n} v = \mathrm{Res}_z Y(u,z)v \frac{(1+z)^{\mathrm{wt}u - 1 + \delta_i(r) + l + r/T}}{z^{2l + \delta_i(r) + \delta_i(T-r)}}. \tag{2.2}$$

Define the linear space $A_{g,n}(V)$ to be the quotient $V/O_{g,n}(V)$. Then $A_{g,n}(V)$ is the untwisted associative algebra $A_n(V)$ as defined in [DLM2] if $g = 1$ and is $A_g(V)$ in [DLM1] if $n = 0$.

We also define a second product $*_{g,n}$ on V for u and v as above:

$$u *_{g,n} v = \sum_{m=0}^{l} (-1)^m \binom{m+l}{l} \mathrm{Res}_z Y(u,z) \frac{(1+z)^{\mathrm{wt}\, u+l}}{z^{l+m+1}} v \tag{2.3}$$

if $r = 0$ and $u *_{g,n} v = 0$ if $r > 0$. Extend linearly to obtain a bilinear product on V.

Lemma 2.1 *If $r \neq 0$ then $V^r \subset O_{g,n}(V)$.*

Proof: Let $v \in V^r$ be homogeneous. Then $v \circ_{g,n} \mathbf{1} \in O_{g,n}(V)$. From the definition we know that

$$v \circ_{g,n} \mathbf{1} = \sum_{j=0}^{\infty} \binom{\mathrm{wt} v - 1 + l + \delta_i(r) + r/T}{j} v_{j-2l-\delta_i(r)-\delta_i(T-r)} \mathbf{1}.$$

Note that $v_k\mathbf{1} = 0$ and $v_{-k-1}\mathbf{1} = \frac{1}{k!}L(-1)^k v$ for $k \geq 0$. Using $L(-1)v \equiv -L(0)v$ modulo $O_{g,n}(V)$ we see that $v \circ_{g,n} \mathbf{1} \equiv \sum_{j=0}^{2l-1+\delta_i(r)+\delta_i(T-r)} \frac{a_j}{j!} \frac{r^j}{T^j} v$ where a_j are integers and $a_{2l-1+\delta_i(r)+\delta_i(T-r)} = \pm 1$. Thus $v \circ_{g,n} \mathbf{1} \equiv cv$ modulo $O_{g,n}$ for a nonzero constant c. This shows that $v \in O_{g,n}(V)$. $\square$

Lemma 2.2 *(i) Assume that $u \in V$ is homogeneous, $v \in V$ and $m \geq k \geq 0$. Then*

$$\mathrm{Res}_z Y(u,z) v \frac{(1+z)^{\mathrm{wt} u - 1 + l + \delta_i(r) + \frac{r}{T} + k}}{z^{2l+\delta_i(r)+\delta_i(T-r)+m}} \in O_{g,n}(V).$$

(ii) For homogeneous $u, v \in V^0$, $u *_n v - v *_n u - \mathrm{Res}_z Y(u,z) v (1+z)^{\mathrm{wt} u - 1} \in O_{g,n}(V)$.

Proof: The proof of (i) is similar to that of Lemma 2.1.2 of [Z]. (ii) follows from a result in Lemma 2.1 (iii) of [DLM2] that $u *_{g,n} v - v *_{g,n} u - \mathrm{Res}_z Y(u,z) v (1+z)^{\mathrm{wt} u - 1} \in O_{1,l}(V^0)$ and the containment $O_{1,l}(V^0) \subset O_{g,n}(V)$. $\square$

Lemma 2.3 *(i) $O_{g,n}(V)$ is a 2 sided ideal of V under $*_{g,n}$.*

(ii) If $I = O_{g,n} \cap V^0$ then $I/O_{1,l}(V^0)$ is a two-sided ideal of $A_{1,l}(V^0)$.

Proof: Since V^r $(r > 0)$ is a subset of $O_{g,n}$ by Lemma 2.1, we see that $O_{g,n}(V) = I \oplus (\oplus_{r=1}^{T-1} V^r)$. Clearly $V^0 *_{g,n} V^r \subset V^r$. So (i) and (ii) are equivalent. We prove (ii). Choose $c \in V^0$ homogeneous and $u \in I$. Using Lemma 2.2 (i) and the argument used to prove Proposition 2.3 of [DLM1] we show that both $c *_{g,n} u$ and $u *_{g,n} c$ lie in $O_{g,n}(V)$. $\square$

The first main result is the following:

Theorem 2.4 *(i) The product $*_{g,n}$ induces the structure of an associative algebra on $A_{g,n}(V)$ with identity $\mathbf{1} + O_{g,n}(V)$.*

(ii) The linear map

$$\phi : v \mapsto e^{L(1)}(-1)^{L(0)}v$$

induces an anti-isomorphism $A_{g,n}(V) \to A_{g^{-1},n}(V)$.

(iii) $\omega + O_{g,n}(V)$ is a central element of $A_{g,n}(V)$.

Proof: (i) follows from the result in [DLM2] that $A_{1,l}(V^0)$ is an associative algebra with respect to $*_{g,n}$ and Lemma 2.3 (i). The proof of (ii) is similar to that of Theorem 2.4 (ii) of [DLM1]. (iii) follows from Theorem 2.3 (iii) of [DLM2] which says that $\omega + O_{1,l}(V^0)$ is a central element of $A_{l,1}(V^0)$ and Lemma 2.3 (ii). $\square$

Proposition 2.5 *The identity map on V induces an onto algebra homomorphism from $A_{g,n}(V)$ to $A_{g,n-1/T}(V)$.*

Proof: If $n = l + i/T$ with $i \geq 1$ then both $A_{g,n}$ and $A_{g,n-1/T}$ are quotients of $A_{1,l}(V^0)$. Otherwise $i = 0$ and $A_{g,n}$ is a quotient algebra of $A_{1,l}(V^0)$ and $A_{g,n-1/T}$ is a quotient algebra of $A_{1,l-1}(V^0)$. By Proposition 2.5 of [DLM2] the identity map induces an epimorphism from $A_{1,l}(V^0)$ to $A_{1,l-1}(V^0)$. So it is enough to show that $O_{g,n}(V) \cap V^0 \subset O_{g,n-1/T}(V) \cap V^0$. But this follows from Lemma 2.2 (i) immediately. $\square$

As in [DLM2], Proposition 2.5 in fact gives us an inverse system $\{A_{g,n}(V)\}$. Denote by $I_{g,n}(V)$ the inverse limit $\varprojlim A_n(V)$. Then

$$I_g(V) = \{a = (a_n + O_{g,n}(V)) \in \prod_{n \geq 0, n \in \frac{1}{T}\mathbb{Z}} A_{g,n}(V) | a_n - a_{n-1/T} \in O_{n-1/T}(v)\}. \qquad (2.4)$$

An interesting problem is to determine $I_g(V)$ explicitly and to study the representations of $I_g(V)$.

3 The Functor Ω_n

Recall from [DLM1] the Lie algebra $V[g]$

$$V[g] = \mathcal{L}(V,g)/D\mathcal{L}(V,g)$$

where

$$\mathcal{L}(V,g) = \oplus_{r=0}^{T-1} t^{r/T}\mathbb{C}[t,t^{-1}]\otimes V^r$$

and by $D = \frac{d}{dt}\otimes 1 + 1\otimes L(-1)$. In order to write down the Lie bracket we introduce the notation $a(q)$ which is the image of $t^q\otimes a \in \mathcal{L}(V,g)$ in $V[g]$. Let $a\in V^r$, $v\in V^s$ and $m,n\in\mathbb{Z}$. Then

$$[a(m+\frac{r}{T}), b(n+\frac{s}{T})] = \sum_{i=0}^{\infty}\binom{m+\frac{r}{T}}{i} a_i b(m+n+\frac{r+s}{T}-i).$$

In fact $V[g]$ is $\frac{1}{T}\mathbb{Z}$-graded Lie algebra by defining the degree of $a(m)$ to be $\mathrm{wt}v - m - 1$ if v is homogeneous. Denote the homogeneous subspace of degree m by $\hat{V}[g]_m$. In particular, $\hat{V}[g]_0$ is a Lie subalgebra.

By Lemma 2.2 (ii) we have

Proposition 3.1 *Regarded $A_{g,n}(V)$ as a Lie algebra, the map $v(\mathrm{wt}v-1)\mapsto v+O_{g,n}(V)$ is a well-defined onto Lie algebra homomorphism from $\hat{V}[g]_0$ to $A_{g,n}(V)$.*

For a module W for the Lie algebra $V[g]$ and a nonnegative $m\in\frac{1}{T}\mathbb{Z}$ we let $\Omega_m(W)$ denote the space of "m-th lowest weight vectors," that is

$$\Omega_m(W) = \{u\in W | V[g]_{-k}u = 0 \ if \ k\geq m\}. \tag{3.1}$$

Then $\Omega_m(W)$ is a module for the Lie algebra $V[g]_0$.

Note that for a g-twisted weak V-module M the map $v(m)\mapsto v_m$ for $v\in V$ and $m\in\frac{1}{T}\mathbb{Z}$ gives a representation of $V[g]$ on M [DLM2]. For a homogeneous $v\in V$ we set $o_p(v) = v_{\mathrm{wt}v-1-p}$ on M.

Lemma 3.2 *Let M be a weak V-module. Then for any homogeneous $u\in V^r$, $v\in V^s$, $p\in\frac{r}{T}+Z, q\in\frac{s}{T}+Z$ with $p\geq q\geq -n$ and $p+q\geq 0$ these exists a unique $w_{u,v}^{p,q}\in V^{r+s}$ such that $o_p(u)o_q(v) = o_{p+q}(w_{u,v}^{p,q})$ on $\Omega_n(M)$. In particular if $s=T-r$ and $p = l+\delta_i(T-r)-k-\frac{r}{T} = -q$ for $k=0,...,l$*

$$w_{u,v}^{p,-p} = \sum_{m=0}^{k}(-1)^m\binom{2l+\delta_i(r)+\delta_i(T-r)-1+m-k}{m}\mathrm{Res}_z Y(u,z)v\frac{(1+z)^{\mathrm{wt}u+l-1+\delta_i(r)+\frac{r}{T}}}{z^{2l+\delta_i(r)+\delta_i(T-r)-k+m}}.$$

The proof is similar to that of Theorem 3.2 of [DLM2]. This lemma is important in constructing admissible g-twisted modules from $A_{g,n}(V)$-modules in the next section.

Theorem 3.3 *Suppose that M is a weak V-module. Then there is a representation of the associative algebra $A_{g,n}(V)$ on $\Omega_n(M)$ induced by the map $a \mapsto o(a) = a_{\mathrm{wt}a-1}$ for homogeneous $a \in V$.*

Proof: By Theorem 5.1 of [DLM2], the map $a \mapsto o(a) = a_{\mathrm{wt}a-1}$ for homogeneous $a \in V^0$ induces a representation of $A_{1,l}(V^0)$ on $\Omega_n(M)$. So it is enough to show that $o(a) = 0$ for $a \in O_{g,n}(V) \cap V^0$. It is clear that $o(L(-1)u + L(0)u) = 0$. It remains to show that $o(u \circ_{g,n} v) = 0$ for $u \in V^r$ and $v \in V^{T-r}$.

Recall identity (10) from [DL]: for $p \in \mathbb{Z}$ and $s, t \in \mathbb{Q}$,

$$\sum_{m\geq 0} (-1)^m \binom{p}{m} (u_{p+s-m}v_{t+m} - (-1)^p v_{p+t-m}u_{s+m}) = \sum_{m\geq 0} \binom{s}{m} (u_{p+m}v)_{s+t-m}. \tag{3.2}$$

Now take $-p = 2l+\delta_i(r)+\delta_i(T-r)$, $s = \mathrm{wt}u-1+l+\delta_i(r)+\frac{r}{T}$, $t = \mathrm{wt}v-1+l+\delta_i(T-r)+\frac{T-r}{T}$. Then on $\Omega_n(M)$ we have $u_{s+m} = v_{t+m} = 0$ for $m \geq 0$. Thus on $\Omega_n(M)$,

$$\begin{aligned} 0 &= \sum_{m\geq 0} \binom{s}{m} (u_{p+m}v)_{s+t-m} \\ &= o(\mathrm{Res}_z Y(u,z)v \frac{(1+z)^{\mathrm{wt}u-1+\delta_i(r)+l+r/T}}{z^{2l+\delta_i(r)+\delta_i(T-r)}}) \\ &= o(u \circ_{g,n} v). \end{aligned}$$

This completes the proof. $\square$

Let $M = \oplus_{m\geq 0, m\in \frac{1}{T}\mathbb{Z}} M(m)$ be an admissible g-twisted module with $M(0) \neq 0$.

Proposition 3.4 *The following hold*

(i) $\Omega_n(M) \supset \oplus_{i=0}^n M(i)$. If M is simple then $\Omega_n(M) = \oplus_{i=0}^n M(i)$.

(ii) Each $M(p)$ is an $\hat{V}[g]_0$-module and $M(p)$ and $M(q)$ are inequivalent if $p \neq q$ and both $M(p)$ and $M(q)$ are nonzero. If M is simple then each $M(p)$ is an irreducible $\hat{V}[g]_0$-module.

(iii) Assume that M is simple. Then each $M(i)$ for $i = 0, ..., n$ is a simple $A_{g,n}(V)$-module and $M(i)$ and $M(j)$ are inequivalent $A_{g,n}(V)$-modules.

The proof is similar to that of Proposition 3.4 of [DLM2].

4 The functor L_n

In Section 3 we have shown how to obtain an $A_{g,n}(V)$-module from an admissible g-twisted V-module. We show in this section that there is a universal way to construct an admissible g-twisted V-module from an $A_{g,n}(V)$-module which cannot factor through $A_{g,n-1/T}(V)$. (If it can factor through $A_{g,n-1/T}(V)$ we can consider the same procedure for $A_{g,n-1/T}(V)$.) As in [DLM2], a certain quotient of the universal object is an admissible g-twisted V-module $L_n(U)$ and L_n defines a functor which is a right inverse to the functor $\Omega_n/\Omega_{n-1/T}$ where $\Omega_n/\Omega_{n-1/T}$ is the quotient functor $M \mapsto \Omega_n(M)/\Omega_{n-1/T}(M)$.

Fix an $A_{g,n}(V)$-module U which cannot factor through $A_{g,n-1/T}(V)$. Then it is a module for $A_{g,n}(V)_{Lie}$ in an obvious way. By Proposition 3.1 we can lift U to a module for the Lie algebra $V[g]_0$, and then to one for $P_n = \oplus_{p>n} V[g]_{-p} \oplus V[g]_0$ by letting $V[g]_{-p}$ act trivially. Define

$$M_n(U) = \mathrm{Ind}_{P_n}^{V[g]}(U) = U(V[g]) \otimes_{U(P_n)} U. \tag{4.1}$$

If we give U degree n, the $\frac{1}{T}\mathbb{Z}$-gradation of $V[g]$ lifts to $M_n(U)$ which thus becomes a $\frac{1}{T}\mathbb{Z}$-graded module for $V[g]$. The PBW theorem implies that $M_n(U)(i) = U(V[g])_{i-n}U$.

We define for $v \in V$,

$$Y_{M_n(U)}(v, z) = \sum_{m \in \frac{1}{T}\mathbb{Z}} v(m) z^{-m-1} \tag{4.2}$$

As in [DLM1], $Y_{M(U)}(v, z)$ satisfies all conditions of a week g-twisted V-module except the associativity which does not hold on $M_n(U)$ in general. We have to divide out by the desired relations.

Let W be the subspace of $M_n(U)$ spanned linearly by the coefficients of

$$\begin{aligned} &(z_0 + z_2)^{\mathrm{wt}a - 1 + l + \delta_i(r) + \frac{r}{T}} Y(a, z_0 + z_2) Y(b, z_2) u \\ &\quad - (z_2 + z_0)^{\mathrm{wt}a + -1 + l + \delta_i(r) + \frac{r}{T}} Y(Y(a, z_0) b, z_2) u \end{aligned} \tag{4.3}$$

for any homogeneous $a \in V^r, b \in V$, $u \in U$. Set

$$\bar{M}_n(U) = M_n(U)/U(V[g])W. \tag{4.4}$$

Theorem 4.1 *The space $\bar{M}_n(U) = \sum_{m \geq 0} \bar{M}_n(U)(m)$ is an admissible g-twisted V-module with $\bar{M}_n(U)(0) \neq 0$, $\bar{M}_n(U)(n) = U$ and with the following universal property: for any weak g-twisted V-module M and any $A_{g,n}(V)$-morphism $\phi : U \to \Omega_n(M)$, there is a unique morphism $\bar{\phi} : \bar{M}_n(U) \to M$ of weak g-twisted V-modules which extends ϕ.*

See Theorem 4.1 of [DLM2] for a similar proof.

Let $U^* = \mathrm{Hom}_{\mathbb{C}}(U, \mathbb{C})$ and let U_s be the subspace of $M_n(U)(n)$ spanned by "length" s vectors

$$o_{p_1}(a_1)\cdots o_{p_s}(a_s)U$$

where $p_1 \geq \cdots \geq p_s$, $p_1 + \cdots p_s = 0$, $p_i \neq 0$, $p_s \geq -n$ and $a_i \in V$. Then by PBW theorem $M_n(U)(n) = \sum_{s\geq 0} U_s$ with $U_0 = U$ and $U_s \cap U_t = 0$ if $s \neq t$. Motivated by the results in Lemma 3.2 we extend U^* to $M_n(U)(n)$ inductively so that

$$\langle u', o_{p_1}(a_1)\cdots o_{p_s}(a_s)u\rangle = \langle u', o_{p_1+p_2}(w_{a_1,a_2}^{p_1,p_2})o_{p_3}(a_3)\cdots o_{p_s}(a_s)u\rangle. \tag{4.5}$$

where $o_j(a) = a(\mathrm{wt}a - 1 - j)$ for homogeneous $a \in V$. We further extend U^* to $M_n(U)$ by letting U^* annihilate $\oplus_{i\neq n} M(U)(i)$.

Set

$$J = \{v \in M_n(U) | \langle u', xv\rangle = 0 \text{ for all } u' \in U^*,\ \text{all } x \in U(V[g])\}.$$

We can now state the second main result of this section.

Theorem 4.2 *Space $L_n(U) = M_n(U)/J$ is an admissible V-module satisfying $L_n(U)(0) \neq 0$ and $\Omega_n/\Omega_{n-1/T}(L_n(U)) \cong U$. Moreover L_n defines a functor from the category of $A_{g,n}(V)$-modules which cannot factor through $A_{g,n-1/T}(V)$ to the category of admissible V-modules such that $\Omega_n/\Omega_{n-1/T} \circ L_n$ is naturally equivalent to the identity. Moreover, $L_n(U)$ is a quotient module of $\bar{M}_n(U)$.*

The proof of this theorem is the most complicated one. Fortunately we can following the proof of Theorem 4.2 of [DLM2] step by step with suitable modifications. Again we refer the reader to [DLM2] for details.

The analogue of Theorem 4.9 of [DLM2] (whose proof is easy) is the following.

Theorem 4.3 *L_n and $\Omega_n/O_{n-1/T}$ are equivalences when restricted to the full subcategories of completely reducible $A_{g,n}(V)$-modules whose irreducible components cannot factor through $A_{g,n-1/T}(V)$ and completely reducible admissible g-twisted V-modules respectively. In particular, L_n and $\Omega_n/\Omega_{n-1/T}$ induce mutually inverse bijections on the isomorphism classes of simple objects in the category of $A_{g,n}(V)$-modules which cannot factor through $A_{g,n-1/T}(V)$ and admissible g-twisted V-modules respectively.*

We also have the generalization of Theorem 4.10 of [DLM2] with a similar proof.

Theorem 4.4 *Suppose that V is a g-rational vertex operator algebra. Then the following hold:*

(a) $A_{g,n}(V)$ is a finite-dimensional, semisimple associative algebra.

(b) The functors L_n and $\Omega_n/O_{n-1/T}$ are mutually inverse categorical equivalences between the category of $A_{g,n}(V)$-modules whose irreducible components cannot factor through $A_{g,n-1/T}(V)$ and the category of admissible g-twisted V-modules.

(c) The functors $L_n, \Omega_n/\Omega_{n-1/T}$ induce mutually inverse categorical equivalences between the category of finite-dimensional $A_{g,n}(V)$-modules whose irreducible components cannot factor through $A_{g,n-1/T}(V)$ and the category of ordinary g-twisted V-modules.

As in [DLM2] one expects that if $A_{g,0}(V)$ is semisimple then V is g-rational. We present some partial results which are applications of $A_{g,n}(V)$-theory.

Theorem 4.5 *All $A_{g,n}(V)$ are finite-dimensional semisimple algebras if and only if V is g-rational.*

References

[B] R. E. Borcherds, Vertex algebras, Kac-Moody algebras, and the Monster, *Proc. Natl. Acad. Sci. USA* **83** (1986), 3068-3071.

[DLM1] C. Dong, H. Li and G. Mason, Twisted representations of vertex operator algebras, q-alg/9509005.

[DLM2] C. Dong, H. Li and G. Mason, Vertex operator algebras and associative algebras, q-alg/9612010.

[DL] C. Dong and Z. Lin, Induced modules for vertex operator algebras, *Commu. Math. Phys.* **179** (1996), 157-184.

[FHL] I. Frenkel, Y. Huang and J. Lepowsky, On axiomatic approaches to vertex operator algebras and modules, Memoirs Amer. Math. Soc. **104**, 1993.

[FLM] I. Frenkel, J. Lepowsky and A. Meurman, *Vertex Operator Algebras and the Monster*, Pure and Appl. Math., **Vol. 134**, Academic Press, Boston, 1988.

[Z] Y. Zhu, Modular invariance of characters of vertex operator algebras, *J. Amer, Math. Soc.* **9** (1996), 237-302.